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A Programmed Text

OHM'S LAW AND ELECTRICAL SOURCES

by

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Syracuse University

Contract No. OE 4-10-102
U.S. Office of Education

(3rd Edition)

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OHM'S LAW

In the previous section we introduced Kirchhoff's two laws having to do with the balance of currents at a node and the balance of voltages around a loop. We ignored the branches themselves, temporarily representing them as boxes. We are now ready to consider the branches that make up electric networks.

Consider simultaneously measuring the voltage and the current of an electrical device when it is connected to a source of electrical energy. We will take the source to be a battery. The instruments are the zero-center meters discussed in the last section. Remember their properties: the current reference is from the + marked terminal of the ammeter to the other terminal through the meter. Similarly the voltage reference + is located ______________.

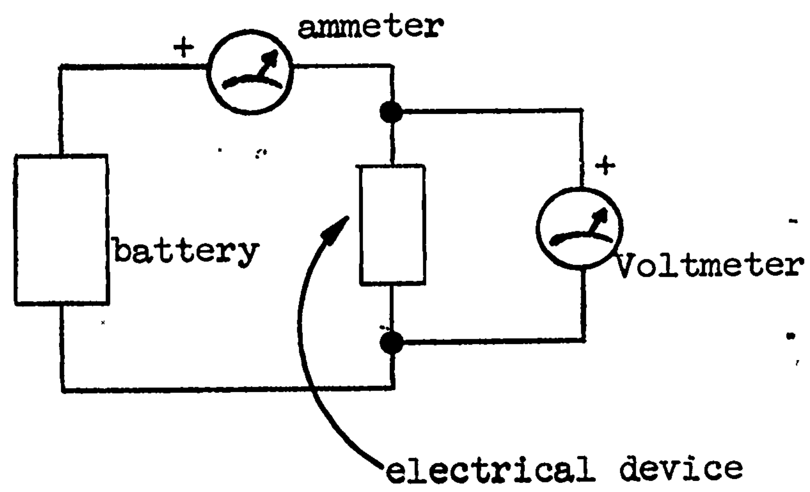


Fig. 1

2

Answer: at the + marked terminal of the voltmeter

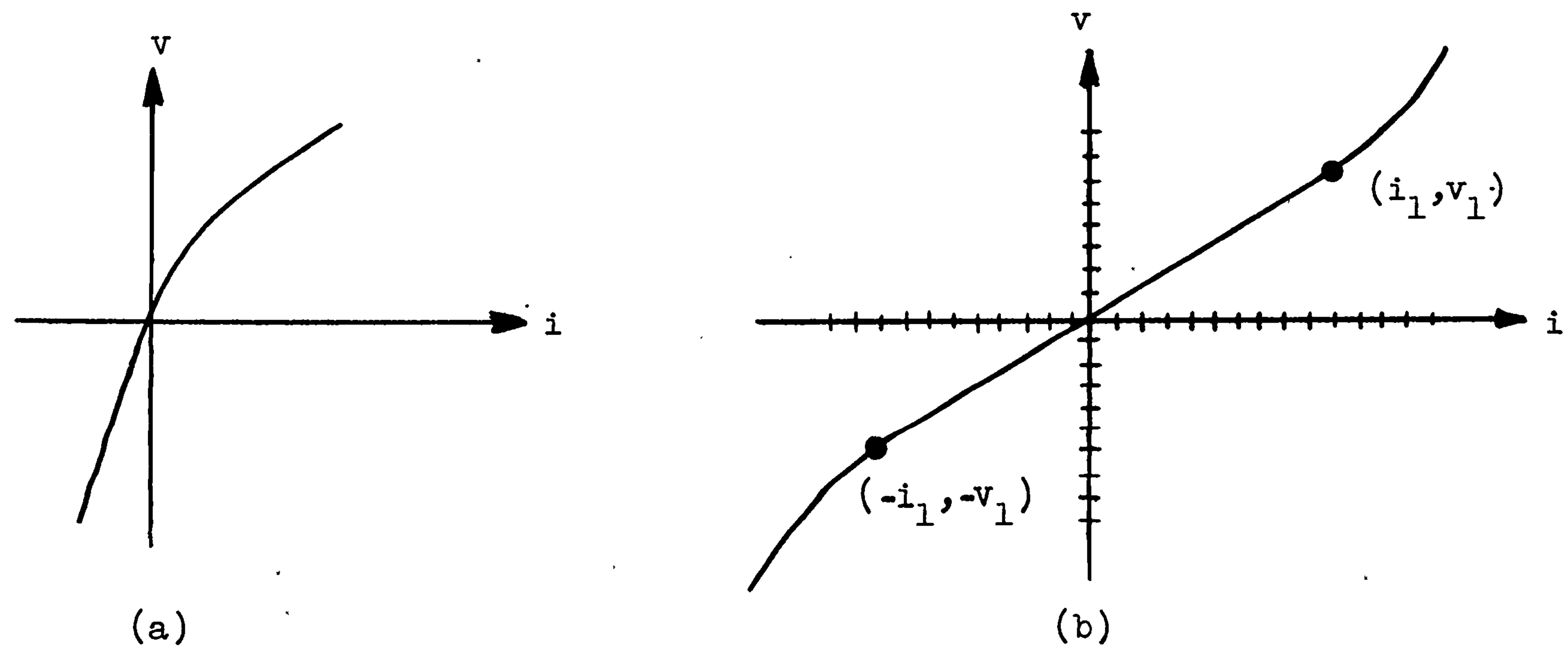


Fig. 2

If the described measurements are made on a large number of different devices, and a graph is made of the voltage plotted against the current for each one, a number of different curves will result.

Two possible curves are shown. Figure 2(a) results when measurements are made on a device called a diode. (Consideration of this will be deferred to a later section.) Other devices, notably metallic objects or lengths of wire, lead to curves like that in Fig. 2(b). Note the characteristics of <u>this</u> curve, Fig. 2(b):

(1) When the current is positive, the voltage is ____________ and when the current is negative the voltage is ___________;

(2) The curve is symmetrical about the origin, in that if v_1 is the voltage for a positive current i_1, the voltage for a current $-i_1$ is ____________________.

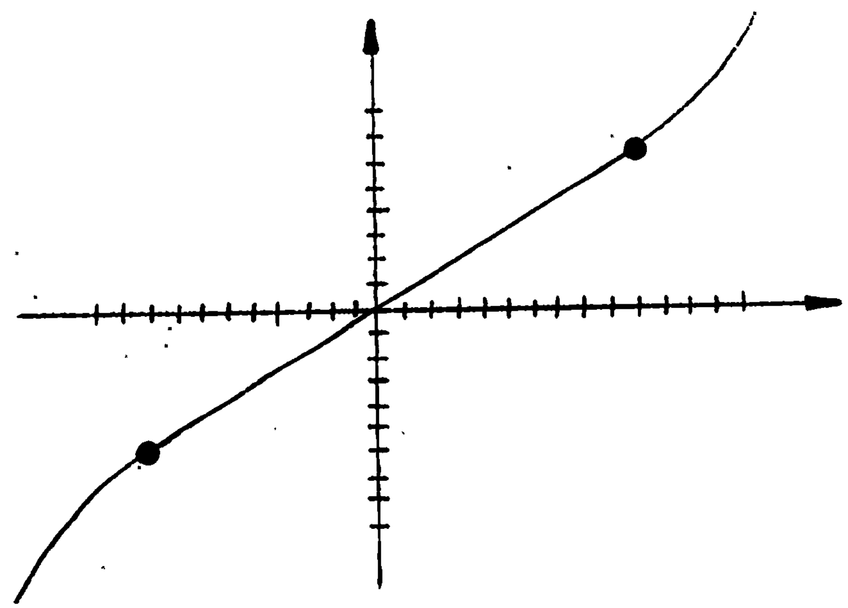
* Note carefully the manner in which the instruments are connected in Fig. 1. We assume further that the instruments are ideal: that is, although current is flowing through the ammeter, there is no voltage across it and although there is a voltage across the voltmeter, there is no current through it.

4

Answer: positive

negative

$- v_1$



Another characteristic of the curve, Fig. 2(b), is that, for a range of positive and negative values around zero current, the curve is approximately a straight line.

Now if the coordinates were x and y instead of i and v, you would no doubt agree that the equation of the straight line could be written as

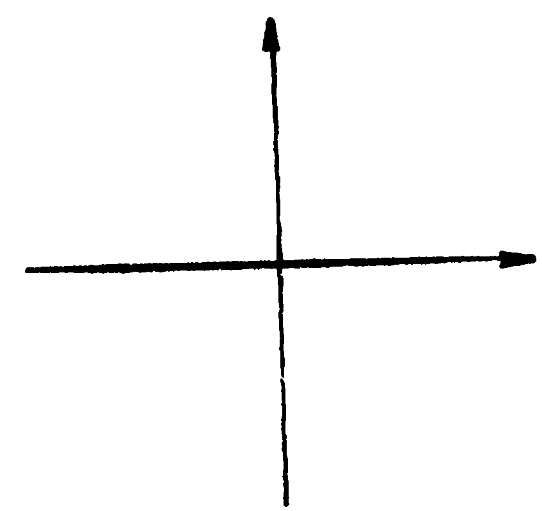
$y =$ ______________________.

6

Answer: $y = mx$, where m is a constant equal to the slope of the line.

This expression is called a <u>linear equation</u>.

From memory, draw the approximately linear v-i curve we have been discussing.



In the range of small current magnitudes the equation relating v to i can be written as ______________________.

8

Answer: $v = Ri$, where R is a constant. (You may have used a different symbol, such as <u>m</u> for the constant R, which is OK; but let's use R from now on.)

The equation $v = Ri$ represents a straight line passing through the origin; it is called a ________________ equation.

10

Answer: linear

Remember that the linear relationship between v and i extends only over a finite range of current. But the linear equation is so simple and easy to handle, we cannot but wish that there were a device having such a voltage-current relationship. We therefore imagine, or mentally <u>invent</u>, a hypothetical device whose v-i relationship we assign to be, or <u>postulate</u> to be, ____________. (Give an equation.)

12

Answer: $v = Ri$

We call this device an <u>ideal resistor</u>, often simply a <u>resistor</u>, and we give it the symbol . By definition, an ideal <u>resistor</u> is

____

____

____.

Answer: A hypothetical (electrical) device postulated to have (or having) the voltage-current relationship $v = Ri$.

NOTE:

Be sure your definition included something about

1. A resistor is assumed to have certain properties, and
2. The equation $v = R_i$ describes a linear relationship.

Do not include anything about the effect of time variation.

Go back and change your definition, as may be appropriate.

We called the hypothetical electrical device having a linear relationship between v and i a <u>resistor</u>. We gave it a schematic symbol consisting of a wavy line. The quantity R, which is the slope of the straight line, is called the <u>resistance</u> of the device. Thus, ________ is the numerical value characterizing a device called a ____________.

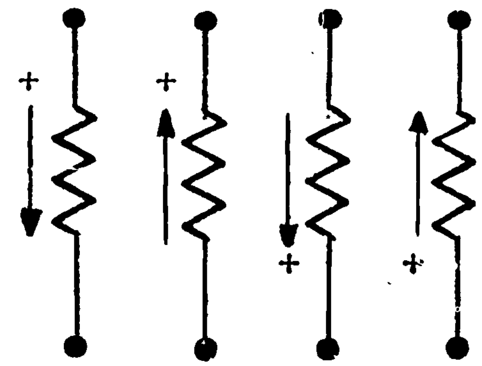
16

Answer: resistance
resistor

Since (with the instruments connected as in Fig. 1) the slope of the straight line is positive, the resistance is a positive quantity.

When a resistor is in a circuit supplied by sources, there will normally be a current through it and a voltage across it. Since there are two alternatives for choosing the voltage reference and two for the current reference, there will be a total of four possible combinations for the voltage and current references together, as shown in the diagram below.

Of these four there are only two distinctly different combinations, as rotating some of them 180 degrees will show. Which pairs are the same?

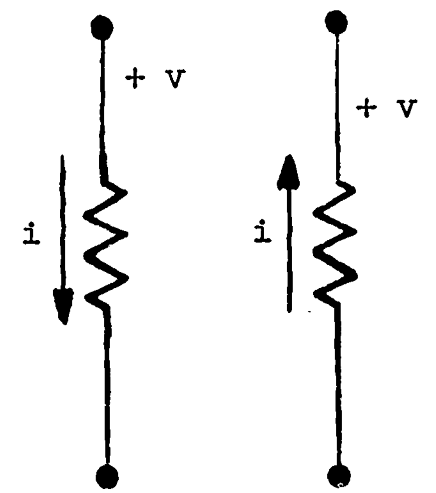


(a) (b) (c) (d)

18

Answer: a and d are the same

b and c are the same



(a) (b)

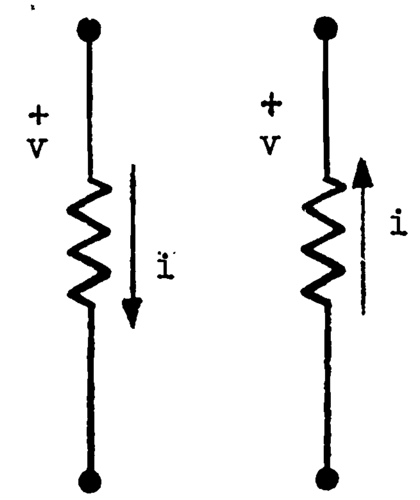
The two distinct choices of reference are, therefore, as shown in the diagram at the left. They can be described as "voltage reference (+) at the tail of current reference (arrow)" and "voltage reference at the tip of the current reference." Remembering the meaning of references relative to meter connections, and the measured v-i curve, the proper expressions relating voltage and current for these two cases are:

A) $v = Ri$ for both . . . . (go to P. 20)

B) $v = Ri$ for (a), $v = -Ri$ for (b) . . . (go to P. 24)

C) $v = -Ri$ for (a), $v = Ri$ for (b) . . . (go to P. 23)

You said $v = Ri$ for both. This is incorrect. Look, if we label the currents in the two cases as i_a and i_b, then $i_b = -i_a$, doesn't it? Well, _if_ for case (b) $v = Ri_b$, then using $i_b = -i_a$, we get $v = -Ri_a$. So, both of them can't be the same; one or the other must carry a minus sign. Turn back to page 19 and try again.



(a) (b)

As we have written Ohm's law, v is expressed in terms of i. It is possible of course, to invert this expression and write $i = \frac{1}{R} v$ which is just another form of Ohm's law. It is convenient to give the reciprocal of R a name; we call 1/R <u>conductance</u> and give it the symbol G: $G = \frac{1}{R}$ and $i = Gv$.

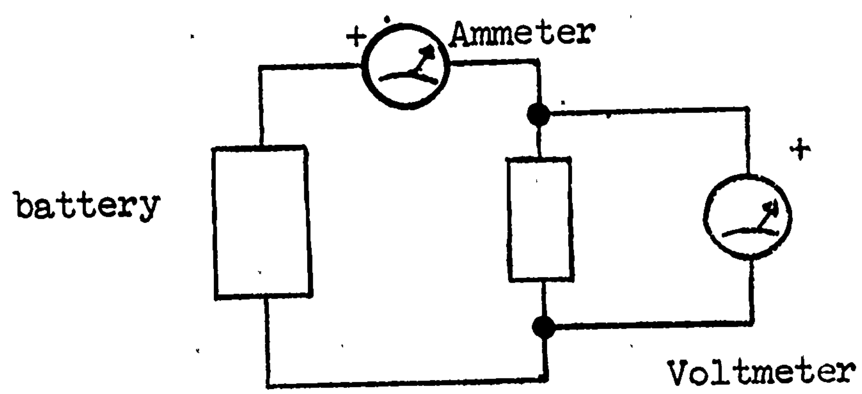
Conductance is the __________ of resistance. The unit in which conductance is measured is obtained by spelling backwards the unit in which resistance is measured; that is, the unit of G is a ____________________.

Answer: reciprocal

mho

(go to page 25)

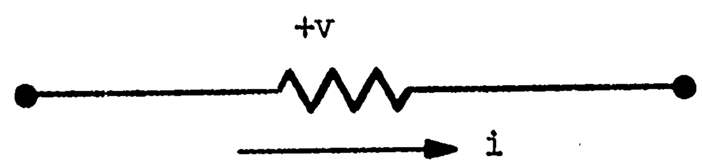
You have them backwards. Look again at the connections of the two meters in Fig. 1 where the measurements were made, and recall how the plus-marked terminals are related to the references of v and i. (Go to P. 24)



$v = Ri$ for (a)
$v = -Ri$ for (b)   That is correct.

The expression $v = Ri$ (or $v = -Ri$, depending on references) is called Ohm's Law, after George Simon Ohm who made measurements on the currents that could be obtained in conducting materials when excited by voltaic cells. It is one of the basic laws in electrical engineering. Although the references for v and i are independent and can be chosen in either one of two ways, it is usually most convenient to choose references as in (a) in order to have Ohm's law written with a positive sign (i.e., $v = Ri$). In honor of George Ohm, the unit of resistance is called the ohm. (Go to page 21)

For the resistor shown there is a voltage $v = 100$ volts when the current is $i = 2$ milliamperes.



The conductance and resistance are:

a) conductance $G =$

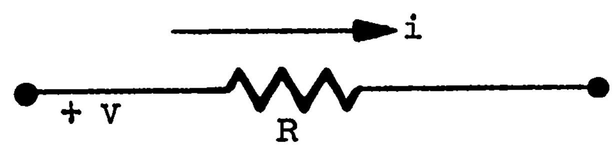
b) resistance $R =$

26

Answer: $G = 2 \times 10^{-5}$ mho or 0.02 millimhos or 20 micromhos.

$R = 50{,}000$ ohms or 50 kilohms.

If the resistance R in the diagram is 100 kilohms and the voltage v is (5 sin 100t) volts, what is the current i? (Express in microamps.)



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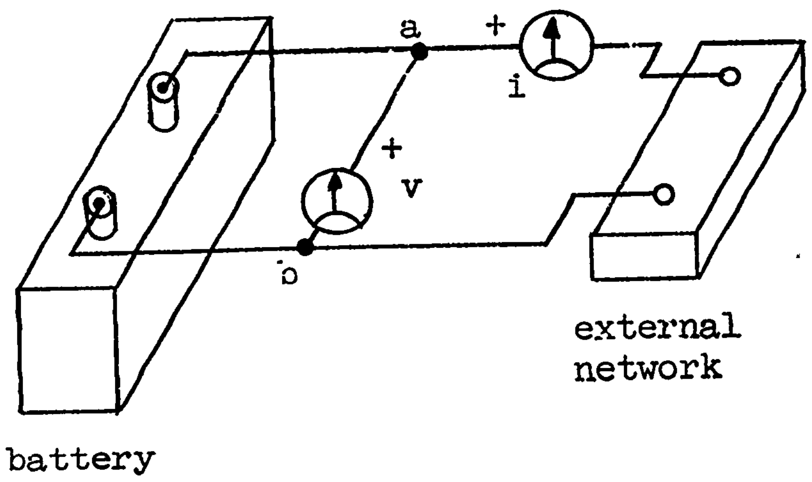
Answer: $i = \frac{v}{R} = 50 \sin 100t$ microamperes.

We have called the <u>hypothetical</u> device having a linear v-i relationship a resistor. But what of the actual physical devices, simultaneous measurements of whose voltage and current give an <u>approximate</u> linear relationship, at least over a limited range of current; what should we call them? We call them <u>resistors</u> also, but whenever there is the possibility of confusion, we qualify the name by the adjective <u>physical</u>. Thus, a __________ ______________ is an actual electrical device and a ____________ ____________ is a hypothetical device having a ________________ voltage-current relationship.



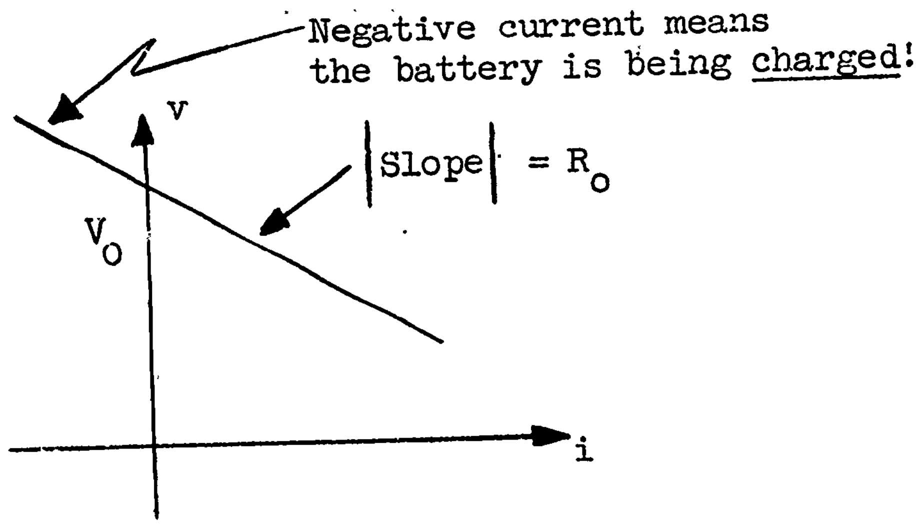


Fig. 3

Answer: <u>physical resistor</u>

<u>ideal resistor</u>

<u>linear</u>

This concludes the introductory discussion of Ohm's law.

SOURCES

There are many devices which convert some other form of energy into electrical energy. The rotating generator converts mechanical energy into electrical; the storage battery converts chemical energy; the photoelectric cell converts light energy; the thermocouple converts heat energy.

If we wanted to design any of these energy conversion devices, it would be necessary to know well the details of the physical processes whereby the energy is converted. But we only want to <u>use</u> these devices in a network. Therefore, we only need the relationship between the voltages and currents at the terminals of the devices.

Consider connecting a battery to an expernal network. The network can be varied so that the voltage and current at the battery terminals can be changed as shown in Fig. 3. Simultaneous measurements of the voltage and current at the terminals of the battery are made (using the ideal instruments described before) and v is plotted against i, as shown. The curve is practically a straight line with intercept V_0 on the voltage axis. Let the magnitude of the slope of the line be called R_0. Write an expression relating v and i from the diagram. v = ____________.

Answer: $v = V_0 - R_0 i$

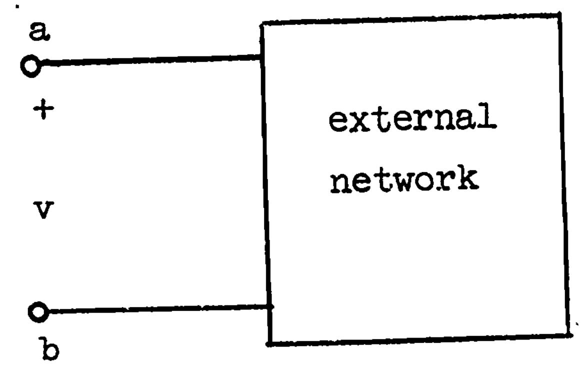


Fig. 4

This expression describes the approximate manner in which the terminal voltage and current of a physical (real, non-ideal) battery are related.

Let's temporarily give $R_o i$ the name v_1 so that the expression becomes $v = V_o - v_1$. This can be considered a statement of Kirchhoff's voltage law around a closed path that has 3 branch voltages. One of these is the terminal voltage of the battery, v, (it is also the voltage across the external network) which is equal to the algebraic sum of the other two branch voltages $(V_o - v_1)$.

Without worrying about the physical nature of the other two branches, draw them in Fig. 4 (using a rectangular box to represent each branch) to form a closed path for which $v = V_o - v_1$.

Label the branches with the appropriate voltages (V_o and v_1) and show both voltage and current references.

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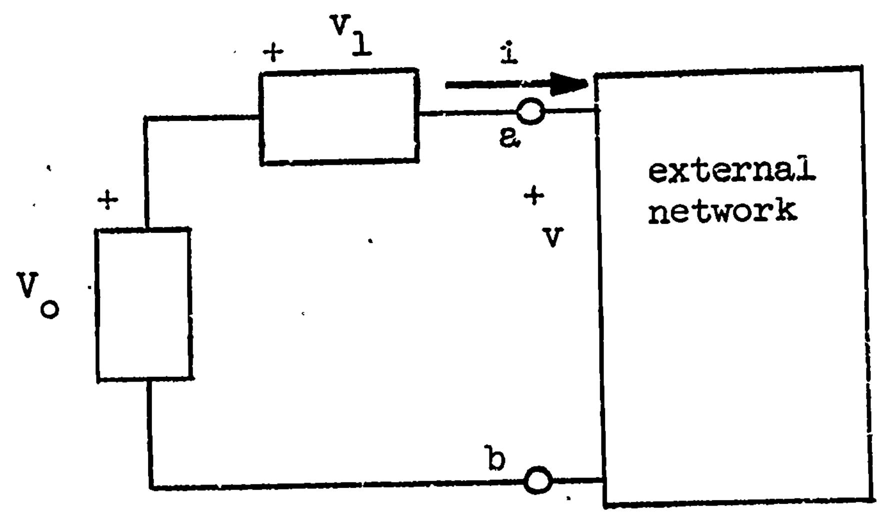
Answer:

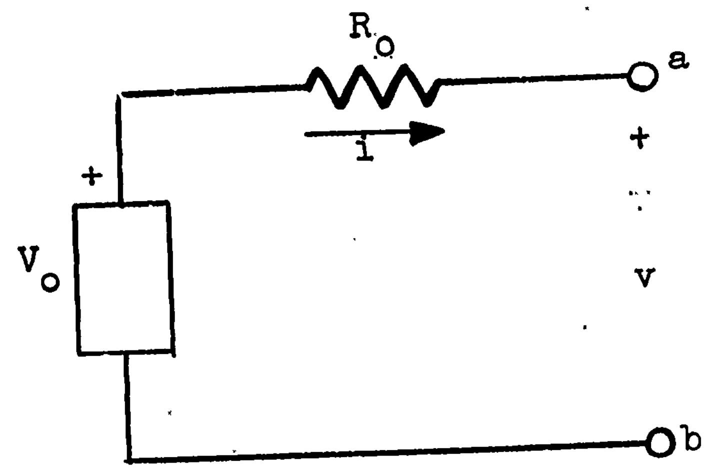


(You may have interchanged the v_1 and V_o boxes, which is OK.) But be sure your voltage reference polarities are correct.)

Since $v_1 = R_o i$, the branch with voltage v_1 across it consists of a __________________. In the space below, redraw the diagram to the left of terminals a-b using an appropriate symbol for this branch and keeping all other symbols.

36

Answer: resistor



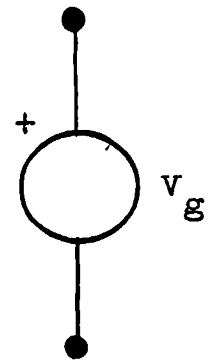
This diagram is a portion of a network having two terminals whose voltage and current are related by $v = V_o - R_o i$. The quantity V_o is a constant, independent of the current. It is the value of the battery terminal voltage when no current is flowing at the terminals -- that is, when the battery is open circuited. (Verify this for yourself from the equation.) It is an idealization of the source of the energy supplied by the battery.

The voltage at the terminals of the battery (v) is dependent on the amount of current through the battery terminals. However, the voltage of the rectangle in the diagram (V_o) is independent of the current. This observation leads us to imagine a hypothetical device which has a voltage across its terminals and has the property at any instant of time that this voltage is fixed and in no way dependent on the current through it. The voltage may change with time (e.g., it might be a sine wave) but this time variation is an internal property of the device and not a function of the current. We shall call such a device a voltage source. (For emphasis we sometimes say an ideal voltage source.) To repeat, a voltage source is ______________. (Give a definition).

Answer: A voltage source is an idealized electrical device whose terminal voltage is independent of its terminal current, although it might depend on time.

(Note: This definition of a voltage source is a general one, and applies whether the voltage is constant or variable with time.)

Instead of a rectangle, which is a symbol used for any branch in a network, we use the circular symbol, shown in the diagram, to represent a voltage source. The voltage reference is an integral part of this diagrammatic representation. (The subscript on v_g stands for "generator".)



The following are claimed to be the voltages of voltage sources:

a) $v_g = 10$
b) $v_g = 8e^{-t}$, $t > 0$ (where t is time)
c) $v_g = 3\ i$, (where i is the current through the source).

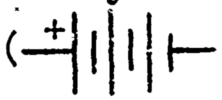
State for each one whether it is possible for the given quantity to be the voltage of a voltage source.

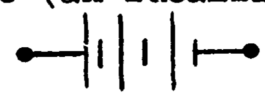
Answer: (a) Yes, a constant voltage

(b) Yes, a voltage which varies with time

(c) No, because it depends on its terminal current

The definition of a voltage source just given is a general one and applies whether the source voltage is constant or variable with time.

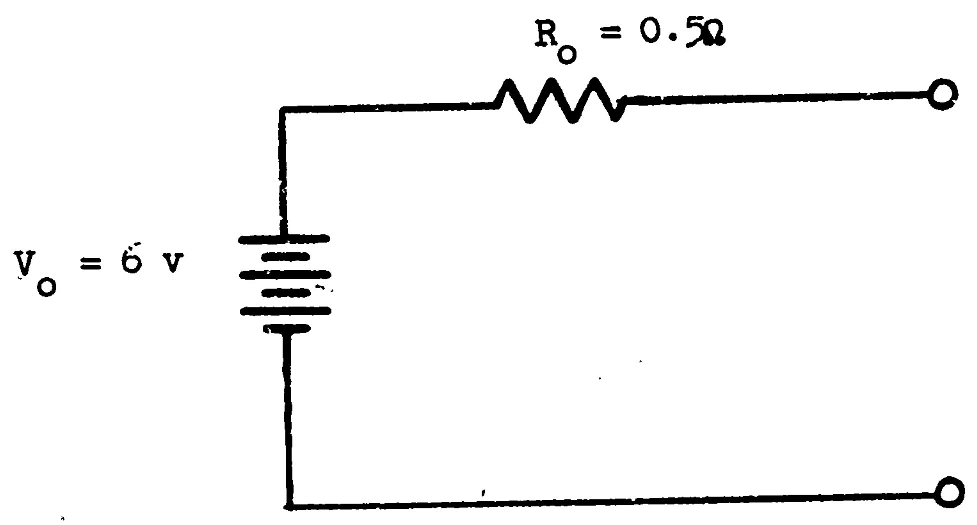
The battery, which has been under discussion, has its own symbol for v_g (or V_o) consisting of a number of alternately long and short lines ().

Real batteries consist of v_g (or V_o) and R_o, called the <u>internal resistance</u> of the battery. If the internal resistance of a battery is zero (an idealized situation), its diagram reduces to the voltage source alone. The symbol  represents an ideal voltage source with a constant voltage. The terminal behavior of a battery is approximated by this ideal voltage source which is hence a <u>model</u> (an electrical model) of the battery. A more accurate model of the battery, of course, consists of the ideal voltage source together with the internal resistance.

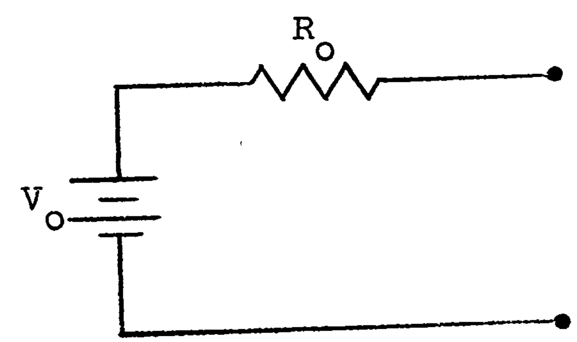
Draw an electrical model (diagram) of a storage battery with an open circuit voltage of 6 volts and an internal resistance of half an ohm. Label all parts.

42

Answer:



The model of a battery we have been discussing, shown here again, is said to be <u>equivalent</u> to, or to be an <u>equivalent network</u> of (often an <u>equivalent circuit</u> of) the physical battery. It is made up of two hypothetical devices:



a (an) ______________________ and
a (an) ______________________.

44

Answer: voltage source
resistor (or ideal resistor)

This model of a battery is equivalent to a battery because the [illegible]al voltage-current relationship for the battery and for this network are [illegible] same. As far as behavior in a network is concerned, then, two devices are equivalent if ____
____.

Answer: the terminal voltage-current relationships of the two devices are identical. As far as the total network is concerned, the two equivalent devices act the same.

To be more precise, a device and its "model" are equivalent only over the range of current and voltage measured. The combination of an ideal voltage source V_o and an internal resistance R_o will have the voltage-current relationship $v = V_o - R_o i$ for any value of current and will retain this relationship no matter how long it is used.

You may have had enough experience with real batteries (flashlights, radios, etc.) to know whether or not this is true of <u>real</u> batteries. Does a physical battery behave like its model? Give reasons for your answer.

Answer: No, it will not. First, the equivalence can only apply over the range we have measured; for different currents the curve may depart from a straight line. Secondly, in a physical battery a finite amount of chemical energy is stored. The battery can convert only as much as was initially stored. Eventually it will be exhausted. But even before this happens, it will deteriorate and the value of its open circuit voltage will decrease and its internal resistance will increase.

A 45 volt dry cell (battery) is providing 2 amperes to an external resistor across which the voltage is 35 volts. What is the value of the internal resistance of the dry cell? R_o = __________________.

Answer: $R_o = 5$ ohms

(Voltage across internal resistance is $45 - 35 = 10$ volts; hence, by Ohm's law, $R_o = 10/2 = 5$ ohms.)

The model of a battery under discussion was arrived at from measurements of terminal voltage and current. The voltage was plotted as ordinate with current as the abscissa. It is, of course, possible to interchange these and plot i against v, as shown in Fig. 5. This is the same as the previous plot with the axes interchanged. The slope of the line is now negative and equal to $1/R_o$ in magnitude.

Suppose the line is extended to intercept the i axis at the value I_o: From the diagram write the equation relating v and i, this time with i taken as the dependent variable.

i = ______________.

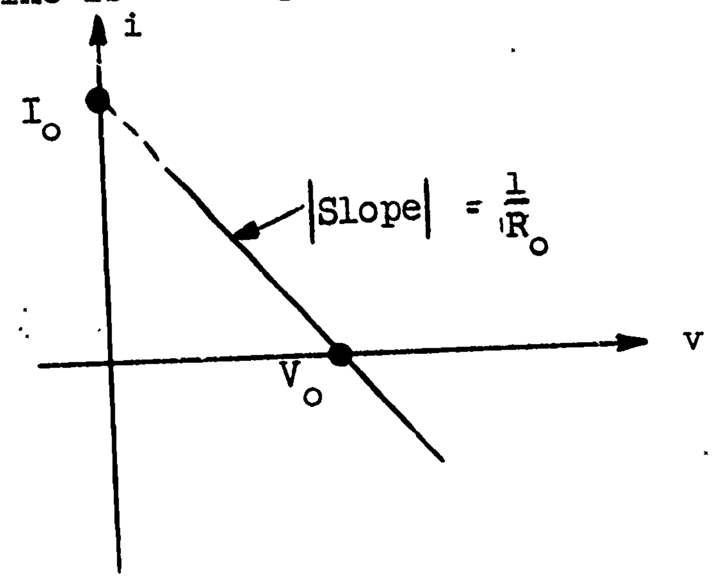
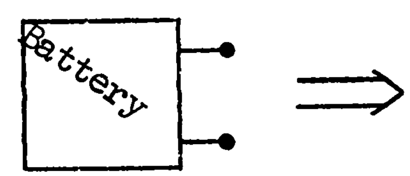


Fig. 5

Answer: $i = I_o - \frac{1}{R_o} v$

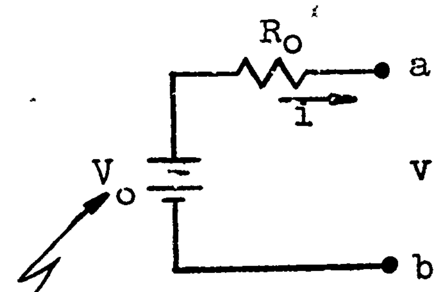
Draw an equivalent circuit for a physical battery and identify the branches by name and symbol.



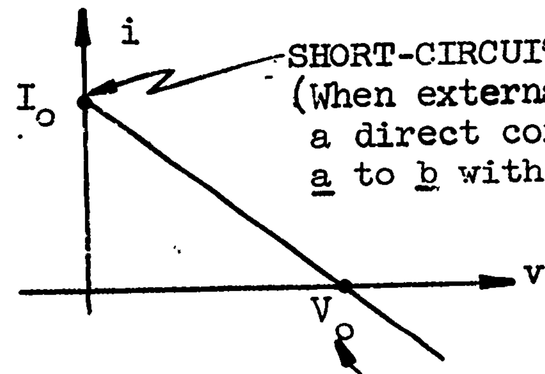
Draw the v-i plot for the equivalent circuit and identify the two intercepts by symbol and name (using terms such as SHORT CIRCUIT and OPEN CIRCUIT).

52(b)

Answer:



$V_o = v_{ab}$ = open circuit voltage (when disconnected from an external network and $i = 0$.)



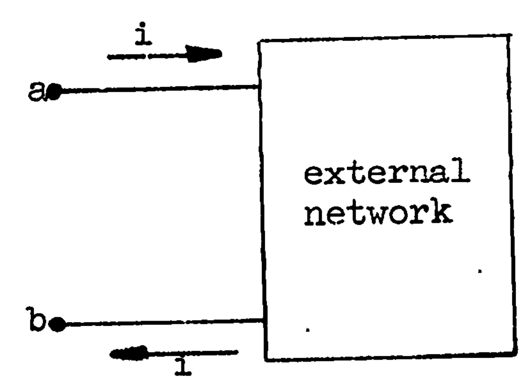
SHORT-CIRCUIT CURRENT (When external network is a direct connection from a to b with ZERO resistance.)

OPEN-CIRCUIT VOLTAGE

If v/R_o is temporarily called i_1, the expression becomes $i = I_o - i_1$.

This looks like a statement of Kirchhoff's current law at a node at which three branches are connected. One of these is the external network in which there is a current i.

Using rectangles for branches, draw the other two branches. We'll discuss the meaning of these branches later. Label the branches with the appropriate currents (including references) and show also the terminal voltage v. (Note that the reference for i is directed away from node a and toward node b.)



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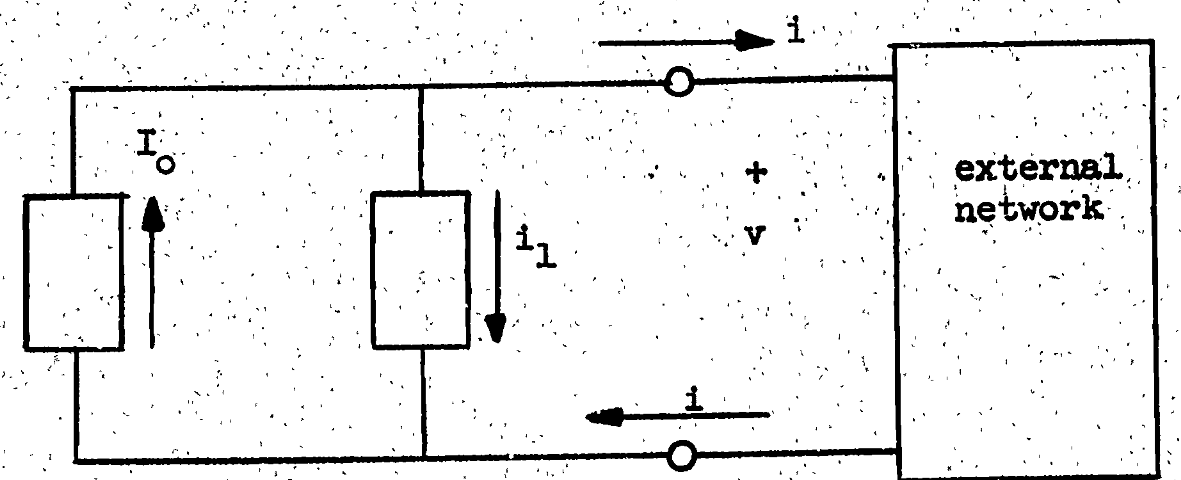
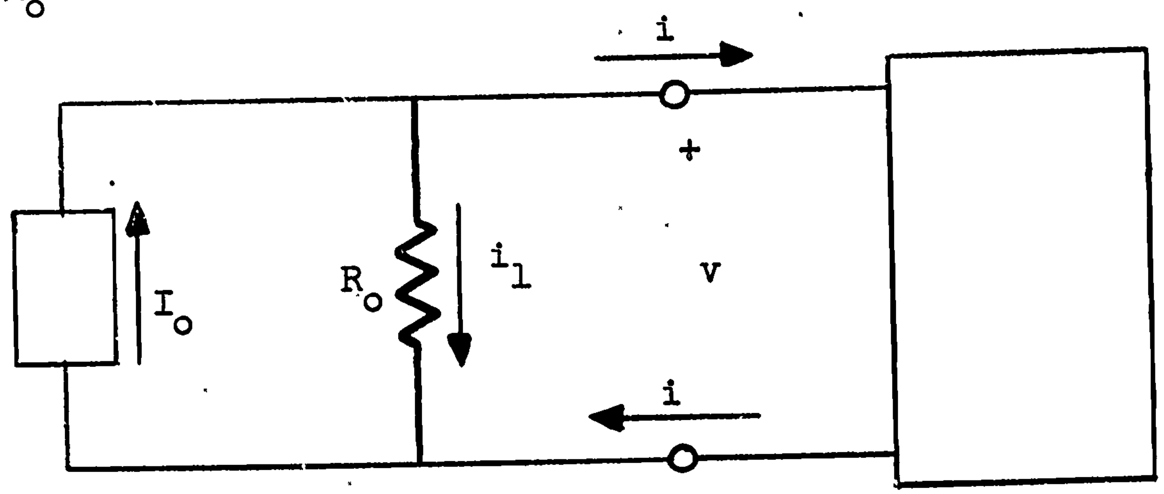
Answer:



Since $i_1 = v/R_o$, in the second branch, and v is the voltage across this branch, the branch in which i_1 is flowing must consist of a ______________ whose value is ____________________.

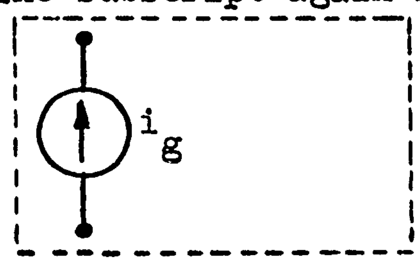
56

Answer: resistor R_o



The third branch carries a constant current I_o which is independent of the voltage at the terminals of the battery, although the terminal current of the battery is certainly dependent on the voltage. We met an analogous situation before when the voltage source was introduced.

This observation leads us now to imagine a hypothetical device which has a current in its terminals with the property that at any instant of time this current is independent of the voltage across the terminals. The current may change with time but this variation is not dependent on the terminal voltage. We shall call such a device a current source (or an ideal current source, for emphasis). It also will be given a circular simbol to designate a source. It can be distinguished from a voltage source by means of the reference which, in this case, is an arrow either alongside the symbol or inside the circle, as shown. (The subscript again stands for generator.)



The equivalent circuit representing the battery then takes the following form. (Complete the diagram and label all parts.)

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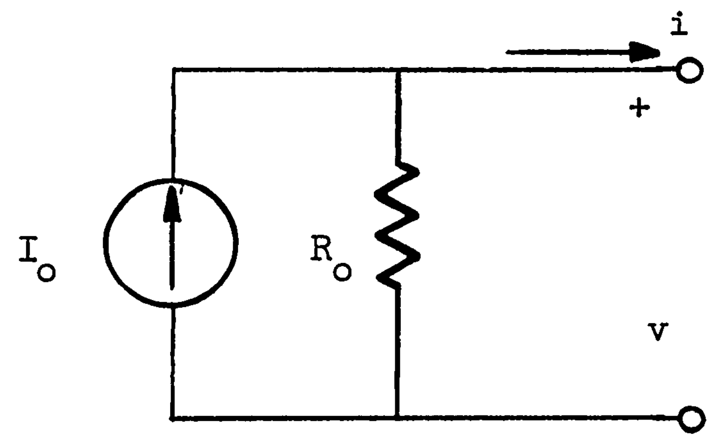
Answer:



(Note: Be sure you distinguish between this symbol for a current source and the symbol for an instrument,  .)

This diagram constitutes an alternative model of a battery. It is <u>equivalent</u> to a battery because it has the same ______________________________.

Describe what you would <u>do</u> at the terminals in order to observe (or use ideal instruments to measure) V_o. How would you measure I_o?

a) To measure V_o:

b) To measure I_o:

Answer: terminal voltage-current relationship.

a) Disconnect any external network and the terminal voltage will be V_o ($i=0$)

b) Connect a short-circuit (zero resistance) across the terminals, then $i = I_o$. (No current will flow through R_o!)

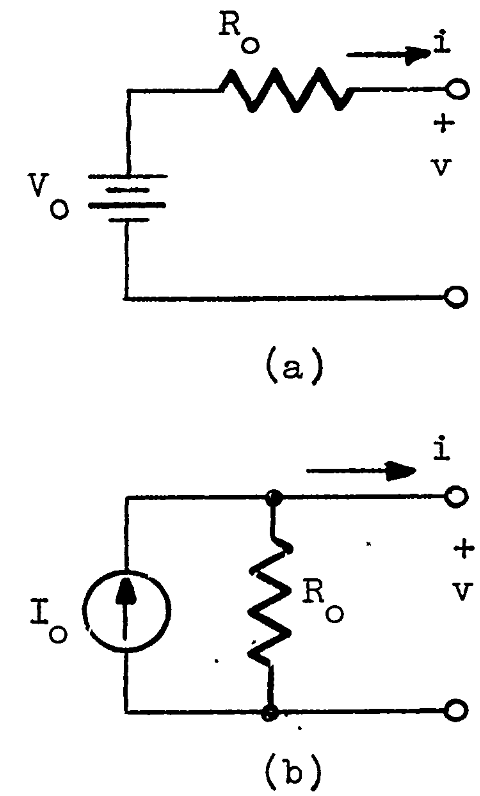


Fig. 6

The two equivalent networks of a battery which we have discussed are shown in Fig. 6. The resistance R_o is the same in both networks. The terminal v-i relationships are obtained by using Kvl in the first and Kcl in the second. Together with Ohm's law for R_o the two expressions are:

(a) $v =$ ____________

(b) $i =$ ____________

Answer: $v = V_o - R_o i$

$$i = I_o - \frac{v}{R_o}$$

The second of these can be solved for v. The result is $v =$ ________________.

Answer: $v = R_oI_o - R_oi$ or $R_o(I_o - i)$

This is to be compared with $v = V_o - R_o i$. Since the two networks are both equivalent to a battery, they should be equivalent to each other. This means they should have the same terminal v-i relationship. Comparing the two expressions leads to the conclusion that equivalence requires $V_o =$ ______________ or $I_o =$ ______________.

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Answer: $V_o = R_o I_o$

$$I_o = \frac{V_o}{R_o}$$

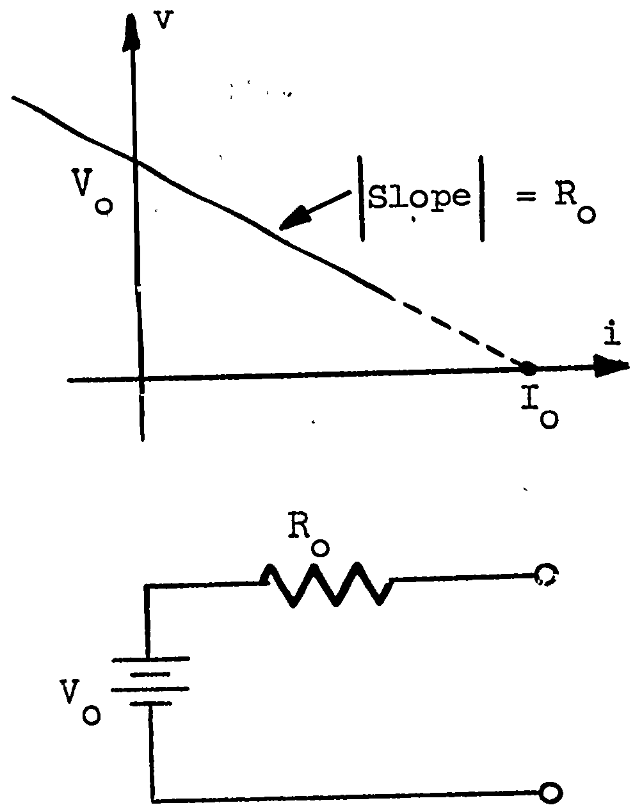


Fig. 7

This conclusion is consistent with the original v-i curve of the battery (extended to intercept the i axis as shown in Fig. 7) from which it is clear that the ratio of V_o to I_o is the (negative) slope R_o.

It has already been noted that V_o is the terminal voltage on open circuit, that is, when nothing is connected to the terminals. Similarly, from the equivalent network it can be seen that I_o is the terminal current when

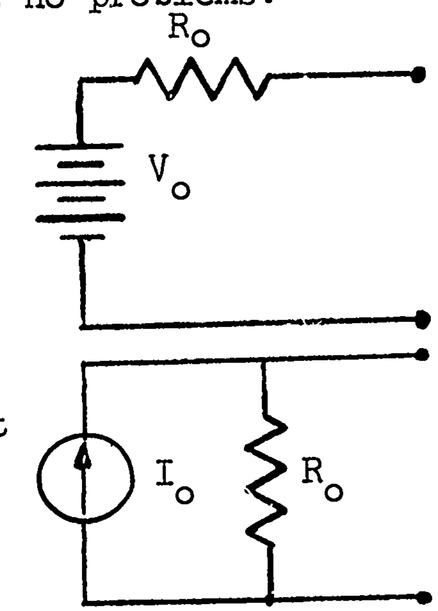
________________________.

Answer: ... when there is a short circuit, that is when the terminals are connected directly together.

Remark

In the case of a physical battery, it is inadvisable to short circuit it, since the battery will then "run down" rapidly. However, as far as the model is concerned, short circuiting the terminals causes no problems.

Each of the two networks shown is equivalent to a battery over the specific ranges of terminal voltage and current that were measured in arriving at the models. However, the two networks are equivalent to each other over all values of terminal voltage and current. This distinction -- an equivalent network of a physical device on the one hand, and two networks which are equivalent to each other on the other hand -- should be kept in mind. Specifically, it is possible to obtain conditions in an equivalent network which are not possible in the physical device of which the network is a model.



In the two equivalent networks of the battery we have been discussing, three quantities appear in both networks. Name these and write their mathematical symbols.

Symbols	Name	Description
1.		
2.		
3.		

Answer: (a) the internal resistance - R_o

(b) the open circuit voltage - V_o

(c) the short-circuit current - I_o

These three quantities are related. If two of them are known the third one can be calculated. Which two? Give an expression for finding the third.

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Answer: <u>Any two</u>. The relationship among them is $V_o = R_o I_o$.

To sum up, we have defined two <u>ideal</u> devices which are sources of electric energy:

(a) ____________________

(b) ____________________.

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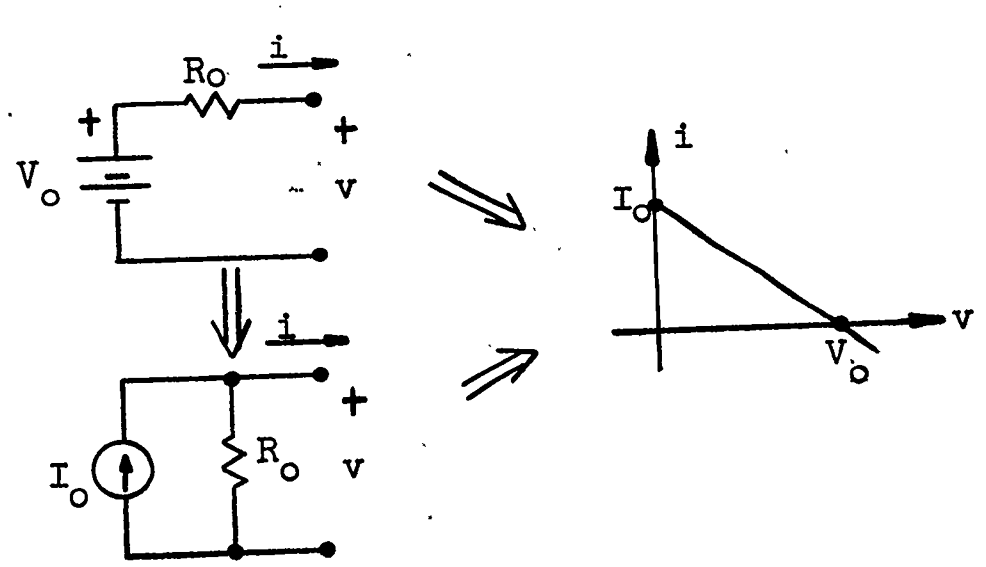
Answer: (a) a voltage source
(b) a current source

By examining the voltage-current relationship at the terminals of a battery, we have found two networks which are equivalent to the battery, by which is meant that ____.

Draw the two equivalent circuits of a battery and the v-i plot which is applicable to <u>both</u> circuits.

Answer: The two networks have the same voltage-current relationship at their terminals as the battery.



The voltage source in one of these two networks and the current source in the other are constants.

Is the following statement true? If it is not, then correct it.

However, a current source need not have a constant current nor a voltage source a constant voltage; they can vary with time.

OE-BR

Answer: Yes, it is true. Either can be a constant or time varying. This terminates the section on sources.